Optimization and the Psychology of Human Decision Making

C. Barth, J. Funke
Experimental and Theoretical Psychology

H. Diedam, M. Engelhart, S. Sager
Interdisciplinary Center for Scientific Computing

HPSC 2009
Hanoi




Outline

Introduction

Mathematical formulation

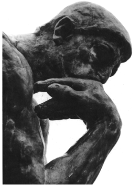
Reformulations

Using Optimization as an Analysis Tool

Algorithm

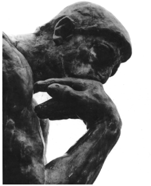
Conclusions and Outlook

Goals of psychologists



- Research complex problem solving of human beings

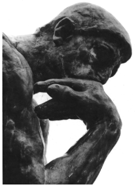
Goals of psychologists



- Research complex problem solving of human beings

- Want to understand how external factors influence thinking
- Example: positive or negative feedback
- Example: stress
- Example: learning effects

Goals of psychologists



- Research complex problem solving of human beings

- Want to understand how external factors influence thinking
- Example: positive or negative feedback
- Example: stress
- Example: learning effects

- Approach: use computer-based test scenarios
- Evaluate performance and correlate it to attributes
- Example: proband's capacity of emotion regulation

Complex problem solving

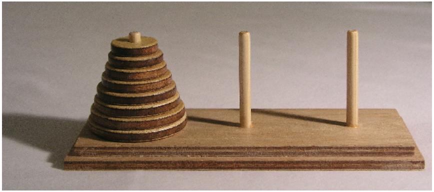
- High-order cognitive process
- Complexity stems from:
  coupling, nonlinearities, dynamics, intransparency, ...
- Psychologists work since ≈ 100 years on understanding

Complex problem solving

- High-order cognitive process
- Complexity stems from:
  coupling, nonlinearities, dynamics, intransparency, ...
- Psychologists work since ≈ 100 years on understanding
- [Ewert&Lambert,1932]: disk problem

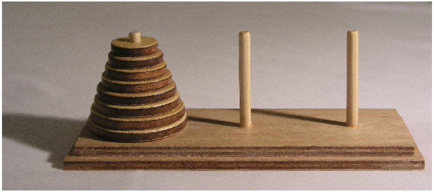
Complex problem solving

- High-order cognitive process
- Complexity stems from:
  coupling, nonlinearities, dynamics, intransparency, ...
- Psychologists work since ≈ 100 years on understanding
- [Ewert&Lambert,1932]: disk problem



Complex problem solving

- High-order cognitive process
- Complexity stems from:
  coupling, nonlinearities, dynamics, intransparency, . . .
- Psychologists work since ≈ 100 years on understanding
- [Ewert&Lambert,1932]: disk problem



- Since 70s/80s: also use computer simulations

Measure capacity to solve complex problems

- Measure proband's performance
  - Performance in a round based test scenario
  - Tailorshop developed in the 80s by Dörner
  - Referenced in many studies and books by now

Measure capacity to solve complex problems

- Measure proband's performance
  - Performance in a round based test scenario
  - Tailorshop developed in the 80s by Dörner
  - Referenced in many studies and books by now
- Collect data from probands:
  - Quantified emotions: own statements
  - Standardized tests to classify probands according to groups, e.g., good or poor emotional regulation
  - Quantified emotions: observation of study leader
  - Quantified emotions: video analysis

The tailorshop

- Round based decision making
- How to produce, distribute, and sell shirts

Manufacturing

→

Logistics

→

Sales

- Goal: maximize profit after 12 months

```
            Hier der Zustand Ihres Ladens am Ende von Monat 0
------------------------------------------------------------------------------
Flüssigkapital              :  165775   Gesamtkapital (Bilanz)    :  250691
verkaufte Hemden            :     407   Nachfrage (aktuell)       :     767
Rohmaterial: Preis          :       4   Rohmaterial: im Lager     :      16
fertige Hemden im Lager     :      81   50-Hemden-Maschinen       :      10
Arbeiter für 50er           :       8   100-Hemden-Maschinen      :       0
Arbeiter für 100er          :       0   Reparatur & Service       :    1200
Lohn pro Arbeiter           :    1080   Sozialkosten pro Arbeiter :      50
Preis pro Hemd              :      52   Ausgaben für Werbung      :    2800
Anzahl der Lieferwagen      :       1   Geschäftslage             : Cityrand
Arbeitszufriedenheit  in %:      57.7   Maschinen-Schäden     in %:     5.9
Produktionsausfall    in %:       0.0
------------------------------------------------------------------------------

                          Maßnahmen für Monat 1
------------------------------------------------------------------------------
R = Rohmaterial einkaufen               H = Hemdenpreis ändern
W = Kosten für Werbung ändern           A = Arbeiter einstellen oder entlassen
M = Maschinen (ver)kaufen, tauschen     I = Instandhaltung, Reparatur/Service
L = Lohn pro Arbeiter ändern            S = Sozialkosten pro Arbeiter ändern
G = Geschäftslage wechseln              T = Lieferwagen kaufen oder verkaufen
                   D = Informationen aus der Datenbank
                   E = Ende der Eingriffe für diesen Monat
```

So what is missing?

- Main motivation for simple test scenarios
  - Optimal solution is known
  - Proband's performance is easy to analyze

So what is missing?

- Main motivation for simple test scenarios
  - Optimal solution is known
  - Proband's performance is easy to analyze
- More complex scenarios
  - Optimal solution is NOT known
  - Performance only comparable among probands
  - or isolated indices, e.g., advance in overall capital
  - Hard to say when and what the wrong decisions were

So what is missing?

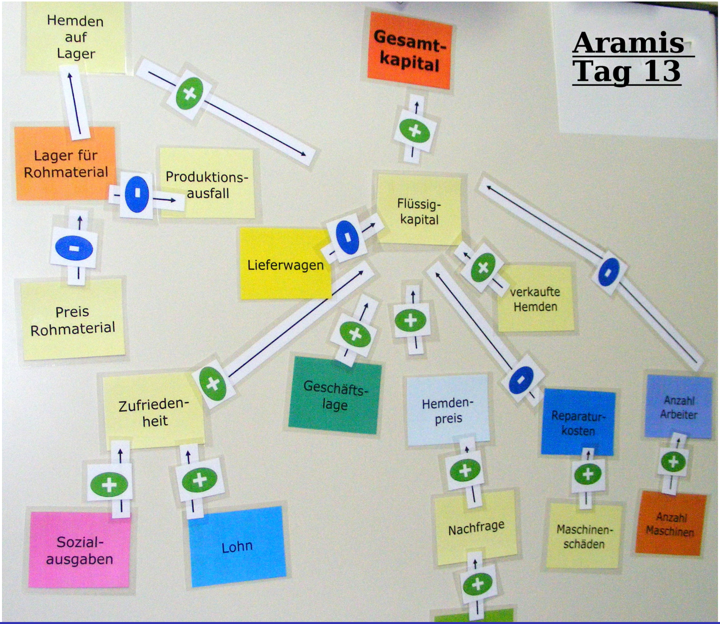
- Main motivation for simple test scenarios
  - Optimal solution is known
  - Proband's performance is easy to analyze
- More complex scenarios
  - Optimal solution is NOT known
  - Performance only comparable among probands
  - or isolated indices, e.g., advance in overall capital
  - Hard to say when and what the wrong decisions were
- Is it possible to have a detailed (and correct) analysis?
- Yes. Need to formulate optimization problem!

Modeling - what was available?

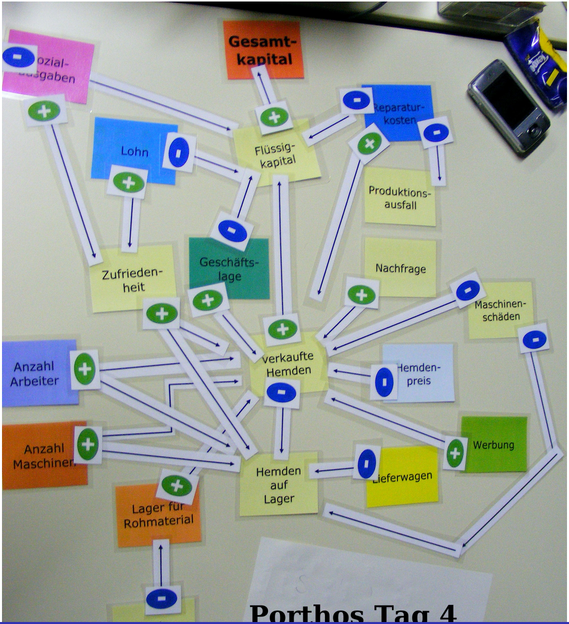
- Heuristic descriptions
- GWBasic source code

GUI used for tests

GUI used for tests



GUI used for tests



Available GW Basic source code – extract

```
2650 ZA=.5+((LO-850)/550)+SM/800:IF ZA>ZM THEN:ZA=ZM
2660 SK=SM*(N1+N2):KA=KA-SK
2670 X=A1:IF N1<X THEN:X=N1
2680 Y=A2:IF N2<Y THEN:Y=N2
2690 PM=X*(MA+RND*4-2)+Y*(MA*2+RND*6-3):PM=PM*(ABS(ZA)^.5)
2700 X=PM:IF RL<X THEN:X=RL
2710 PA=X:HL=HL+PA:RL=RL-PA:KA=KA-(PA*1)-(RL*.5)
2720 NA=(NA/2+280)*1.25*2.7181^(-(PH^2)/4250):KA=KA-HL
2730 X=NA:IF HL<X THEN:X=HL
2740 VH=X:HL=HL-VH:KA=KA+VH*PH
2750 KA=KA-WE
2760 X1=WE/5:IF X1>NM THEN:X1=NM
2770 KA=KA-LW*500:X1=X1+LW*100
2780 KA=KA-GL*2000
2790 X=0:IF GL=.5 THEN:X=.1:ELSE IF GL=1 THEN:X=.2
2800 X1=X1+X1*X
2810 NA=X1+(RND*100-50)
2820 RP=2+(RND*6.5)
2830 MA=MA-.1*MA+(RS/(A1+A2*1E-08))*.017
2840 IF MA>MM THEN:MA=MM
2850 KA=KA-RS
```

Observations

- Nonlinear

```
2720 NA=(NA/2+280)*1.25*2.7181^(-(PH^2)/4250)
```

Observations

- Nonlinear

```
2720 NA=(NA/2+280)*1.25*2.7181^(-(PH^2)/4250)
```

- Integer variables

```
2790 X=0:IF GL=.5 THEN:X=.1:ELSE IF GL=1 THEN:X=.2
```

Observations

- Nonlinear

```
2720 NA=(NA/2+280)*1.25*2.7181^(-(PH^2)/4250)
```

- Integer variables

```
2790 X=0:IF GL=.5 THEN:X=.1:ELSE IF GL=1 THEN:X=.2
```

- Random values ξ

```
2810 NA=X1+(RND*100-50)
```

Observations

- Nonlinear

  ```
  2720 NA=(NA/2+280)*1.25*2.7181^(-(PH^2)/4250)
  ```
- Integer variables

  ```
  2790 X=0:IF GL=.5 THEN:X=.1:ELSE IF GL=1 THEN:X=.2
  ```
- Random values ξ

  ```
  2810 NA=X1+(RND*100-50)
  ```
- Nondifferentiable

  ```
  2650 ZA=.5+((LO-850)/550)+SM/800:IF ZA>ZM THEN:ZA=ZM
  ```

Observations

- Nonlinear

```
2720 NA=(NA/2+280)*1.25*2.7181^(-(PH^2)/4250)
```

- Integer variables

```
2790 X=0:IF GL=.5 THEN:X=.1:ELSE IF GL=1 THEN:X=.2
```

- Random values ξ

```
2810 NA=X1+(RND*100-50)
```

- Nondifferentiable

```
2650 ZA=.5+((LO-850)/550)+SM/800:IF ZA>ZM THEN:ZA=ZM
```

- Sometimes variable time k, sometimes already updated

```
2690 PM=X*(MA+RND*4-2)+Y*(MA*2+RND*6-3):PM=PM*(ABS(ZA)^.5)
2700 X=PM:IF RL<X THEN:X=RL
2710 PA=X:HL=HL+PA:RL=RL-PA:KA=KA-(PA*1)-(RL*.5)
```

Abstract optimization model

- Dynamic model with discrete time $k = 0 \dots N$

Abstract optimization model

- Dynamic model with discrete time $k = 0 \dots N$
- Decisions $u_k = u(k)$ and states $x_k = x(k)$

Abstract optimization model

- Dynamic model with discrete time $k = 0 \dots N$
- Decisions $u_k = u(k)$ and states $x_k = x(k)$
- Given initial values x_0 and parameters p

Abstract optimization model

- Dynamic model with discrete time $k = 0 \dots N$
- Decisions $u_k = u(k)$ and states $x_k = x(k)$
- Given initial values x_0 and parameters p
- Random values ξ

Abstract optimization model

- Dynamic model with discrete time $k = 0 \ldots N$
- Decisions $u_k = u(k)$ and states $x_k = x(k)$
- Given initial values x_0 and parameters p
- Random values ξ
- Goal: find decisions u_k
  to maximize objective function of x_N

Abstract optimization model

- Dynamic model with discrete time $k = 0 \ldots N$
- Decisions $u_k = u(k)$ and states $x_k = x(k)$
- Given initial values x_0 and parameters p
- Random values ξ
- Goal: find decisions u_k
  to maximize objective function of x_N

Abstract optimization model

- Dynamic model with discrete time $k = 0 \dots N$
- Decisions $u_k = u(k)$ and states $x_k = x(k)$
- Given initial values x_0 and parameters p
- Random values ξ
- Goal: find decisions u_k
  to maximize objective function of x_N

$$
\begin{array}{llcll}
\max\limits_{x,u} & F(x_N) & & & \\
\text{s.t.} & x_{k+1} & = & G(x_k, x_{k+1}, u_k, p, \xi), & k = 0 \dots N-1, \\
 & 0 & \leq & H(x_k, x_{k+1}, u_k, p), & k = 0 \dots N-1, \\
 & u_k & \in & \Omega, & k = 0 \dots N-1.
\end{array}
$$

Control functions u_k

Decision	$low \leq$	u_k	$\leq up$
advertisement	$0 \leq$	WE	$\leq \infty$
shirt price	$10 \leq$	PH	≤ 100
buy raw material	$0 \leq$	ΔRL	$\leq \infty$
workers 50	$-A_1 \leq$	ΔA_1	$\leq \infty$
workers 100	$-A_2 \leq$	ΔA_2	$\leq \infty$
buy machines 50	$0 \leq$	ΔM_1	$\leq \infty$
buy machines 100	$0 \leq$	ΔM_2	$\leq \max(0, MA - 35) \cdot \infty$
sell machines 50	$0 \leq$	δM_1	$\leq M_1$
sell machines 100	$0 \leq$	δM_2	$\leq M_2$
maintenance	$0 \leq$	RS	$\leq \infty$
wages	$850 \leq$	LO	$\leq \infty$
social spenses	$0 \leq$	SM	$\leq \infty$
buy vans	$0 \leq$	ΔLW	$\leq \infty$
sell vans	$0 \leq$	δLW	$\leq LW$
Choose site		GL	$\in \{c, r, v\}$

State variables x_{k+1} and x_k

State	x_{k+1}	$G(x_k, x_{k+1}, u_k, p, \xi)$
machines 50	M_1	$M_1 + \Delta M_1 - \delta M_1$
machines 100	M_2	$M_2 + \Delta M_2 - \delta M_2$
workers 50	A_1	$A_1 + \Delta A_1$
workers 100	A_2	$A_2 + \Delta A_2$
demand	NA	$100\xi - 50$
		$+ \left(\min(\frac{WE}{5}, NM) + 100LW\right) \cdot \begin{cases} 1.2 & \text{if } GL = c \\ 1.1 & \text{if } GL = r \\ 1.0 & \text{if } GL = v \end{cases}$
vans	LW	$LW + \Delta LW - \delta LW$
shirts sales	VH	$\min(HL, \frac{5}{4}(\frac{NA}{2} + 280) \cdot 2.7181^{-\frac{PH^2}{4250}})$
shirts stock	HL	$HL + PA - VH$
possible production	PM	$(\min(A_1, M_1)(MA + 4\xi - 2)$
		$+ \min(A_2, M_2)(2MA + 6\xi - 3)) \cdot \lvert ZA\rvert^{\frac{1}{2}}$
actual production	PA	$\min(PM, RL + \Delta RL)$
material price	RP	$2 + 6.5\xi$
material stock	RL	$RL + \Delta RL - PA$
satisfaction	ZA	$\min\left(ZM, \frac{1}{2} + \frac{LO - 850}{550} + \frac{SM}{800}\right)$
machine capacity	MA	$\min\left(MM, 0.9MA + 0.017\frac{RS}{M_1 + 10^{-8}M_2}\right)$

State variables: money

$$
\begin{aligned}
UK &= KA + VH \cdot PH - RP \cdot \Delta RL \\
&\quad -10000\Delta M_1 + 8000\frac{MA}{MM}\delta M_1 - 20000\Delta M_2 + 16000\frac{MA}{MM}\delta M_2 \\
&\quad -SK - WE - RS - (A_1 + A_2) \cdot LO \\
&\quad -PA - \frac{1}{2}RL - (HL + PA) \\
&\quad -10000 \cdot \Delta LW + (8000 - 100k) \cdot \delta LW - 500LW \\
&\quad - \begin{cases} 2000 & \text{if } GL = c \\ 1000 & \text{if } GL = r \\ 500 & \text{if } GL = v \end{cases} \\
KA &= UK \left(1 + \begin{cases} GZ & \text{if } UK \geq 0 \\ SZ & \text{if } UK < 0 \end{cases} \right)
\end{aligned}
$$

Goal: maximize L_N:

$$
\begin{aligned}
L &= KA + \frac{MA}{MM}(8000M_1 + 16000M_2) \\
&\quad +(8000 - 100k) \cdot LW + 2RL + 20HL
\end{aligned}
$$

Fixed initial values x_0 and parameters p

State	x_k	$x_0 =$
machines 50	M_1	10
machines 100	M_2	0
workers 50	A_1	8
workers 100	A_2	0
demand	NA	766.636
material price	RP	3.9936
material stock	RL	16.06787
shirts stock	HL	80.7164
machine capacity	MA	47.04
cash	KA	165774.66
vans	LW	1

Parameter	p	$p =$
maximum demand	NM	900
interest rate	GZ	0.0025
debt rate	SZ	0.0066
maximum machine capacity	MM	50
maximum satisfaction	ZM	1.7

Modeling issues

$$
\begin{aligned}
\max_{x,u} \quad & F(x_N) \\
\text{s.t.} \quad & x_{k+1} = G(x_k, x_{k+1}, u_k, p, \xi), && k = 0 \dots N-1, \\
& 0 \leq H(x_k, x_{k+1}, u_k, p), && k = 0 \dots N-1, \\
& u_k \in \Omega, && k = 0 \dots N-1.
\end{aligned}
$$

- More realistic modeling (delays, memory effects, . . .)

Modeling issues

$$
\begin{aligned}
\max_{x,u} \quad & F(x_N) \\
\text{s.t.} \quad & x_{k+1} = G(x_k, x_{k+1}, u_k, p, \xi), \quad k = 0 \dots N-1, \\
& 0 \leq H(x_k, x_{k+1}, u_k, p), \quad k = 0 \dots N-1, \\
& u_k \in \Omega, \quad k = 0 \dots N-1.
\end{aligned}
$$

- More realistic modeling (delays, memory effects, ...)
- Modeling errors

Modeling issues

$$\max_{x,u} \quad F(x_N)$$

$$\text{s.t.} \quad \begin{array}{lcll} x_{k+1} & = & G(x_k, x_{k+1}, u_k, p, \xi), & k = 0 \ldots N-1, \\ 0 & \leq & H(x_k, x_{k+1}, u_k, p), & k = 0 \ldots N-1, \\ u_k & \in & \Omega, & k = 0 \ldots N-1. \end{array}$$

- More realistic modeling (delays, memory effects, . . .)
- Modeling errors
- Random values ξ

Modeling issues

$$\max_{x,u} \quad F(x_N)$$

$$\text{s.t.} \quad \begin{array}{lcll} x_{k+1} & = & G(x_k, x_{k+1}, u_k, p, \xi), & k = 0 \ldots N-1, \\ 0 & \leq & H(x_k, x_{k+1}, u_k, p), & k = 0 \ldots N-1, \\ u_k & \in & \Omega, & k = 0 \ldots N-1. \end{array}$$

- More realistic modeling (delays, memory effects, . . .)
- Modeling errors
- Random values ξ
- Bounds on variables

Modeling issues

$$\max_{x,u} \quad F(x_N)$$

$$\begin{array}{llll} \text{s.t.} \quad x_{k+1} & = & G(x_k, x_{k+1}, u_k, p, \xi), & k = 0 \dots N-1, \\ 0 & \leq & H(x_k, x_{k+1}, u_k, p), & k = 0 \dots N-1, \\ u_k & \in & \Omega, & k = 0 \dots N-1. \end{array}$$

- More realistic modeling (delays, memory effects, ...)
- Modeling errors
- Random values ξ
- Bounds on variables
- Integer decisions

Modeling issues

$$\begin{aligned}
\max_{x,u} \quad & F(x_N) \\
\text{s.t.} \quad & x_{k+1} = G(x_k, x_{k+1}, u_k, p, \xi), \quad k = 0 \ldots N-1, \\
& 0 \leq H(x_k, x_{k+1}, u_k, p), \quad k = 0 \ldots N-1, \\
& u_k \in \Omega, \quad k = 0 \ldots N-1.
\end{aligned}$$

- More realistic modeling (delays, memory effects, ...)
- Modeling errors
- Random values ξ
- Bounds on variables
- Integer decisions
- $F(\cdot)$, $G(\cdot)$ and $H(\cdot)$ continuously differentiable?
  Expressions including `if`, `min`, or `max` are not!

Consistency

- More realistic model only with new study

Consistency

- More realistic model only with new study
- Modelling errors: have to accept and include them

  $MA = \min\left(MM, 0.9MA + 0.017\frac{RS}{M_1+10^{-8}M_2}\right)$

  $\longrightarrow RS = \epsilon$ optimal

Consistency

- More realistic model only with new study
- Modelling errors: have to accept and include them

  $MA = \min\left(MM, 0.9MA + 0.017\frac{RS}{M_1+10^{-8}M_2}\right)$

  $\longrightarrow RS = \epsilon$ optimal
- Random values ξ

```
140 X=RND(-1)
...
2810 NA=X1+(RND*100-50)
```

Random values ξ can be treated as parameters p!

Integer decisions

Decision	$low \leq$	u_k	$\leq up$
advertisement	$0 \leq$	WE	$\leq \infty$
shirt price	$10 \leq$	PH	≤ 100
buy raw material	$0 \leq$	ΔRL	$\leq \infty$
workers 50	$-A_1 \leq$	ΔA_1	$\leq \infty$
workers 100	$-A_2 \leq$	ΔA_2	$\leq \infty$
buy machines 50	$0 \leq$	ΔM_1	$\leq \infty$
buy machines 100	$0 \leq$	ΔM_2	$\leq \max(0, MA - 35) \cdot \infty$
sell machines 50	$0 \leq$	δM_1	$\leq M_1$
sell machines 100	$0 \leq$	δM_2	$\leq M_2$
maintenance	$0 \leq$	RS	$\leq \infty$
wages	$850 \leq$	LO	$\leq \infty$
social spenses	$0 \leq$	SM	$\leq \infty$
buy vans	$0 \leq$	ΔLW	$\leq \infty$
sell vans	$0 \leq$	δLW	$\leq LW$
Choose site		GL	$\in \{c, r, v\}$

Bounds

- Optimizer's intuition: no bounds on variables
  $\longrightarrow$ unbounded solution

Bounds

- Optimizer's intuition: no bounds on variables
  $\longrightarrow$ unbounded solution
- Combination of model error and no bound. Demand

$$NA = a + \left(\min(\frac{WE}{5}, NM) + 100LW \right) \cdot b$$

  enters into number of shirts sold

$$VH = \min(HL, \frac{5}{4}(\frac{NA}{2} + 280) \cdot 2.7181^{-\frac{PH^2}{4250}})$$

Bounds

- Optimizer's intuition: no bounds on variables
  $\longrightarrow$ unbounded solution
- Combination of model error and no bound. Demand

$$NA = a + \left(\min(\frac{WE}{5}, NM) + 100LW\right) \cdot b$$

  enters into number of shirts sold

$$VH = \min(HL, \frac{5}{4}(\frac{NA}{2} + 280) \cdot 2.7181^{-\frac{PH^2}{4250}})$$

- Need to include bounds – consistency!

Nondifferentiabilities

- $\min\left(ZM, \frac{1}{2} + \frac{LO-850}{550} + \frac{SM}{800}\right)$

Nondifferentiabilities

- $\min\left(ZM, \frac{1}{2} + \frac{LO-850}{550} + \frac{SM}{800}\right) \longrightarrow \frac{1}{2} + \frac{LO-850}{550} + \frac{SM}{800} \leq ZM$

Nondifferentiabilities

- $\min\left(ZM, \frac{1}{2} + \frac{LO-850}{550} + \frac{SM}{800}\right) \longrightarrow \frac{1}{2} + \frac{LO-850}{550} + \frac{SM}{800} \leq ZM$
- $\min(PM, RL + \Delta RL) \longrightarrow RL + \Delta RL \leq PM$
- $\min(HL, \frac{5}{4}(\frac{NA}{2} + 280) \cdot 2.7181^{-\frac{PH^2}{4250}})$
  $\longrightarrow \frac{5}{4}(\frac{NA}{2} + 280) \cdot 2.7181^{-\frac{PH^2}{4250}} \leq HL$
- $\min(\frac{WE}{5}, NM) \longrightarrow \frac{WE}{5} \leq NM$
- $\min\left(MM, 0.9MA + 0.017\frac{RS}{M_1+10^{-8}M_2}\right) \longrightarrow$
  $0.9MA + 0.017\frac{RS}{M_1+10^{-8}M_2} \leq MM$
- $\min(A_1, M_1), \min(A_2, M_2) \longrightarrow A_1 \leq M_1, A_2 \leq M_2$
- Buy machines (100) only if $MA > 35$:
  $\longrightarrow 0 \leq \Delta M_2 \leq \max(0, MA - 35) \cdot \infty$
  $\longrightarrow MA \geq 36$
- $KA = UK \left(1 + \begin{cases} GZ & \text{if } UK \geq 0 \\ SZ & \text{if } UK < 0 \end{cases}\right)$?

Optimization problem

$$\begin{array}{llll} \max\limits_{x,u} & F(x_N) & & \\ \text{s.t.} & x_{k+1} = G(x_k, x_{k+1}, u_k, p), & k = 0 \ldots N-1, \\ & 0 \leq H(x_k, x_{k+1}, u_k, p), & k = 0 \ldots N-1, \\ & u_k \in \Omega, & k = 0 \ldots N-1. \end{array}$$

- 5 continuous control functions

Optimization problem

$$
\begin{array}{llll}
\max\limits_{x,u} & F(x_N) & & \\
\text{s.t.} & x_{k+1} = G(x_k, x_{k+1}, u_k, p), & & k = 0 \dots N-1, \\
& 0 \leq H(x_k, x_{k+1}, u_k, p), & & k = 0 \dots N-1, \\
& u_k \in \Omega, & & k = 0 \dots N-1.
\end{array}
$$

- 5 continuous control functions
- 10 integer control functions

Optimization problem

$$
\begin{aligned}
\max_{x,u} \quad & F(x_N) \\
\text{s.t.} \quad & x_{k+1} = G(x_k, x_{k+1}, u_k, p), \quad k = 0 \dots N-1, \\
& 0 \leq H(x_k, x_{k+1}, u_k, p), \quad k = 0 \dots N-1, \\
& u_k \in \Omega, \quad k = 0 \dots N-1.
\end{aligned}
$$

- 5 continuous control functions
- 10 integer control functions
- 17 state functions

Optimization problem

$$
\begin{array}{llll}
\max\limits_{x,u} & F(x_N) & & \\
\text{s.t.} & x_{k+1} = G(x_k, x_{k+1}, u_k, p), & k = 0 \ldots N-1, \\
& 0 \leq H(x_k, x_{k+1}, u_k, p), & k = 0 \ldots N-1, \\
& u_k \in \Omega, & k = 0 \ldots N-1.
\end{array}
$$

- 5 continuous control functions
- 10 integer control functions
- 17 state functions
- No uncertainty

Optimization problem

$$\max_{x,u} \quad F(x_N)$$

$$\begin{aligned}
\text{s.t.} \quad x_{k+1} &= G(x_k, x_{k+1}, u_k, p), & k = 0 \dots N-1, \\
0 &\leq H(x_k, x_{k+1}, u_k, p), & k = 0 \dots N-1, \\
u_k &\in \Omega, & k = 0 \dots N-1.
\end{aligned}$$

- 5 continuous control functions
- 10 integer control functions
- 17 state functions
- No uncertainty
- Differentiable

Optimization problem

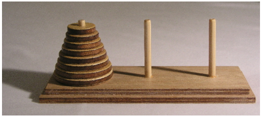
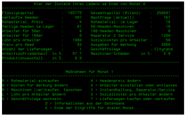
$$\max_{x,u} \quad F(x_N)$$

$$\begin{aligned} \text{s.t.} \quad x_{k+1} &= G(x_k, x_{k+1}, u_k, p), & k = 0 \ldots N-1, \\ 0 &\leq H(x_k, x_{k+1}, u_k, p), & k = 0 \ldots N-1, \\ u_k &\in \Omega, & k = 0 \ldots N-1. \end{aligned}$$

- 5 continuous control functions
- 10 integer control functions
- 17 state functions
- No uncertainty
- Differentiable
- Mixed-integer Nonlinear Program (MINLP)

Intermediate summary

- Go from simple test scenarios to complex scenarios

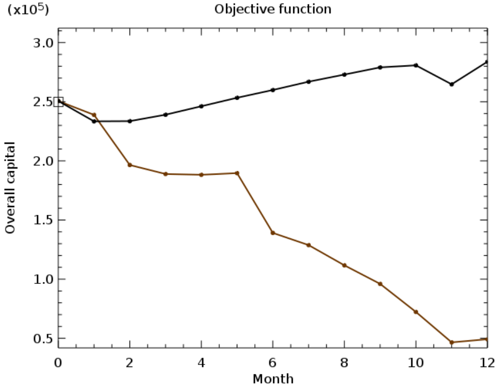

$\longrightarrow$


- Determine month(s) k with **bad** decisions
- Do not use progress in objective as currently done!

- Compare optimal solutions at time k and $k+1$ as measure
- Optimal solutions = solutions of MINLPs

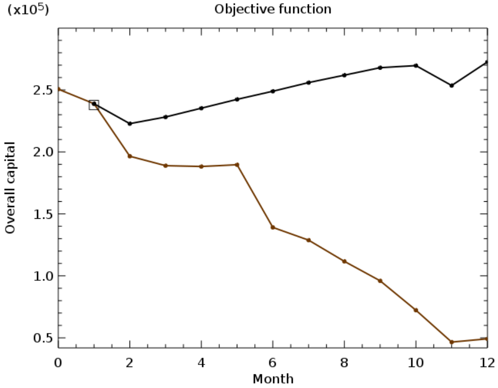
Analysis

- For every data set
  - For every month from 0 to 11
    - Calculate optimal solution for rest of time
    - Store objective value at end time



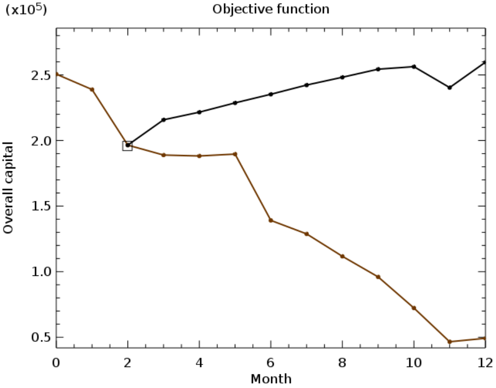
Analysis

- For every data set
  - For every month from 0 to 11
    - Calculate optimal solution for rest of time
    - Store objective value at end time



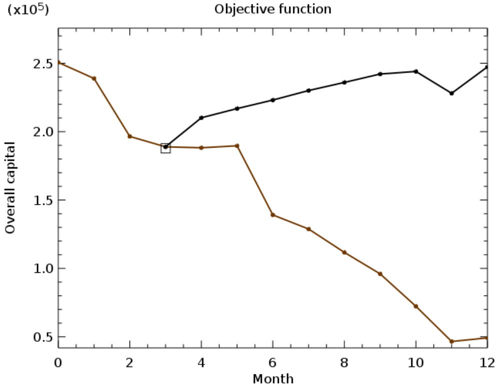
Analysis

- For every data set
  - For every month from 0 to 11
    - Calculate optimal solution for rest of time
    - Store objective value at end time



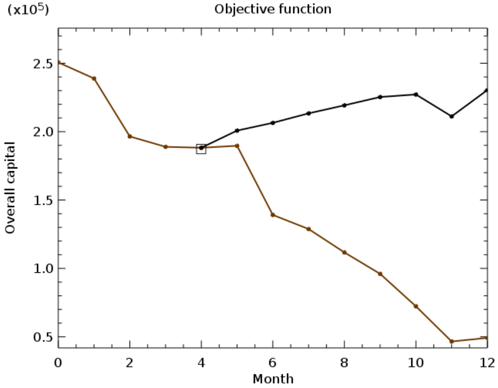
Analysis

- For every data set
  - For every month from 0 to 11
    - Calculate optimal solution for rest of time
    - Store objective value at end time



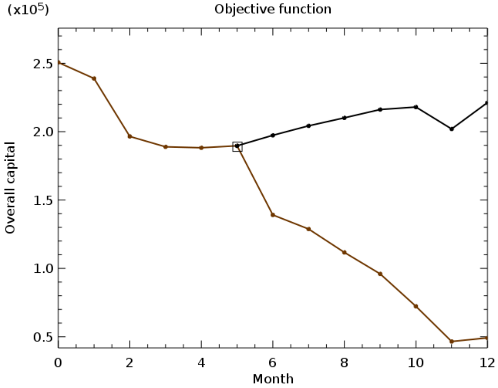
Analysis

- For every data set
  - For every month from 0 to 11
    - Calculate optimal solution for rest of time
    - Store objective value at end time



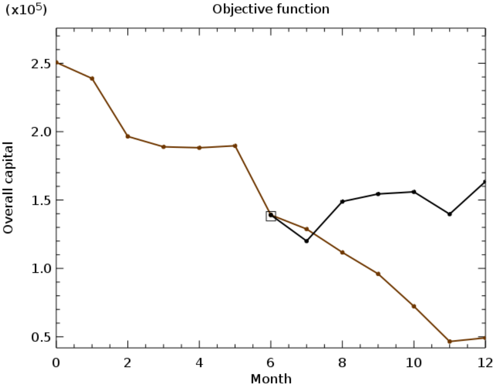
Analysis

- For every data set
  - For every month from 0 to 11
    - Calculate optimal solution for rest of time
    - Store objective value at end time



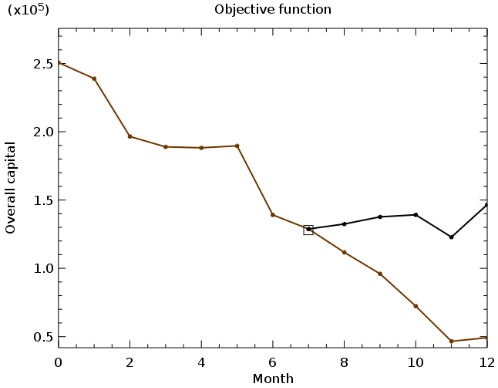
Analysis

- For every data set
  - For every month from 0 to 11
    - Calculate optimal solution for rest of time
    - Store objective value at end time



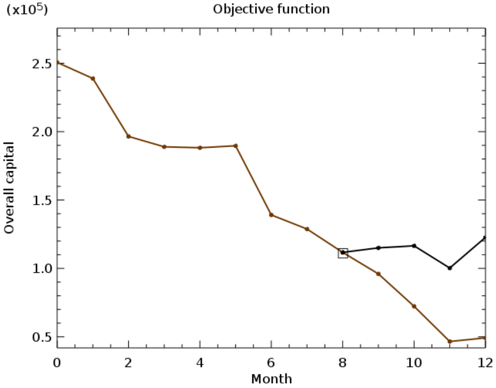
Analysis

- For every data set
  - For every month from 0 to 11
    - Calculate optimal solution for rest of time
    - Store objective value at end time



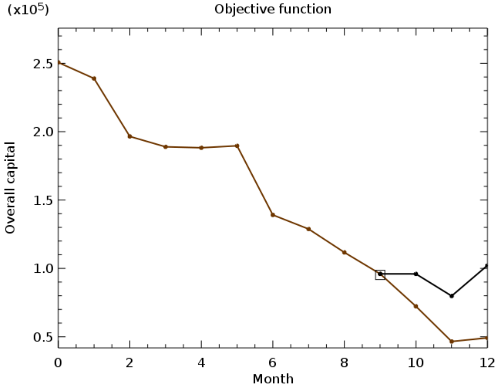
Analysis

- For every data set
  - For every month from 0 to 11
    - Calculate optimal solution for rest of time
    - Store objective value at end time



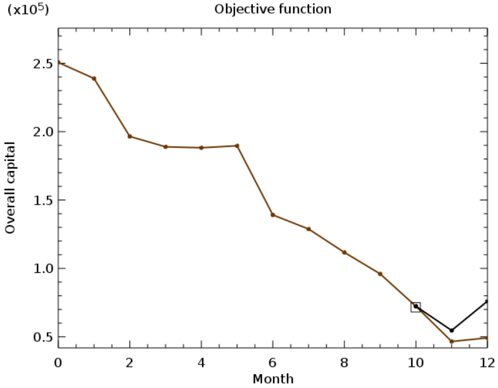
Analysis

- For every data set
  - For every month from 0 to 11
    - Calculate optimal solution for rest of time
    - Store objective value at end time



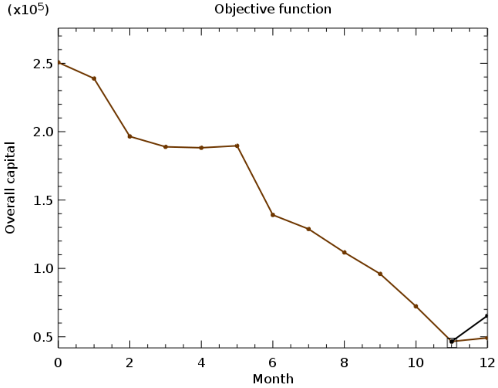
Analysis

- For every data set
  - For every month from 0 to 11
    - Calculate optimal solution for rest of time
    - Store objective value at end time



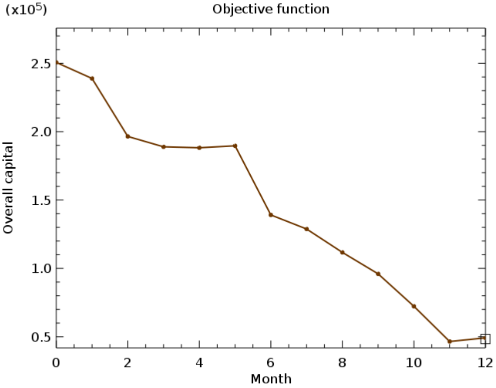
Analysis

- For every data set
  - For every month from 0 to 11
    - Calculate optimal solution for rest of time
    - Store objective value at end time

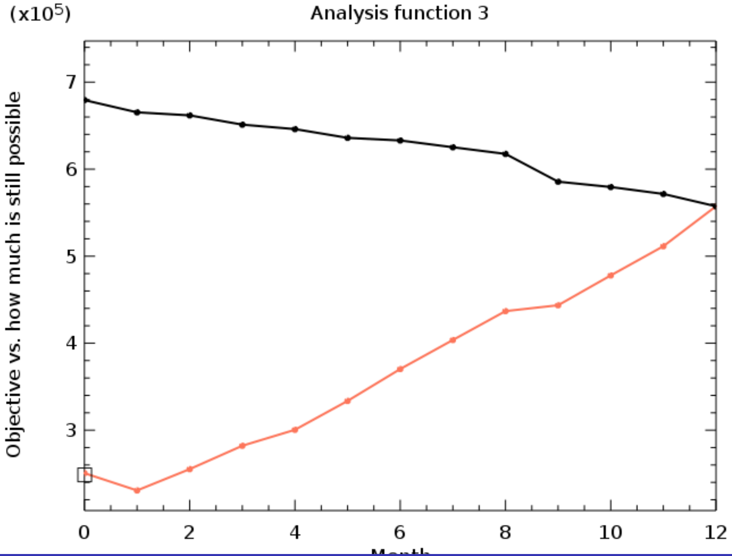


Analysis

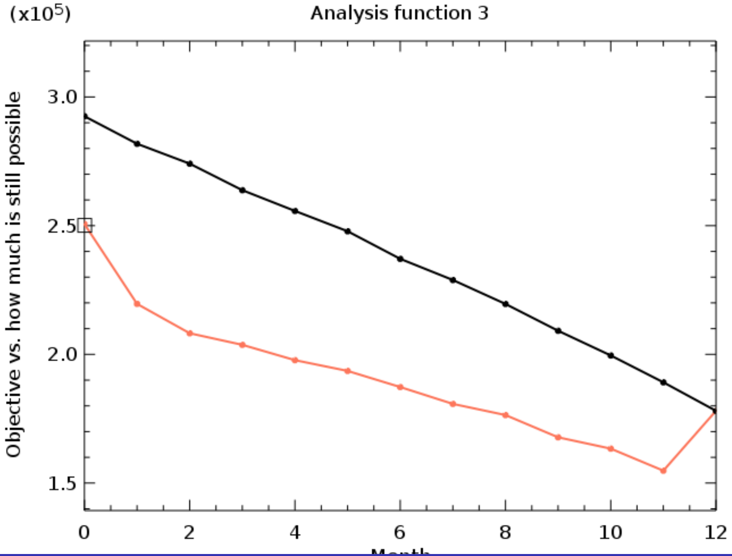
- For every data set
  - For every month from 0 to 11
    - Calculate optimal solution for rest of time
    - Store objective value at end time



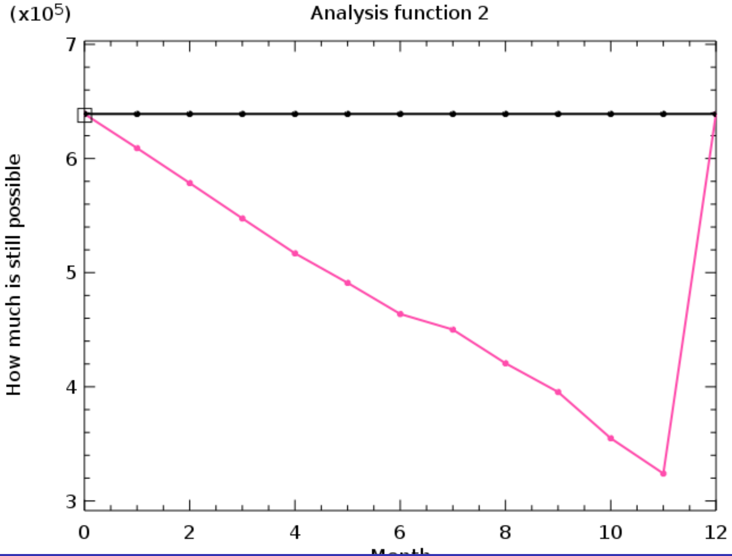
Objective of proband vs. potential (in black)



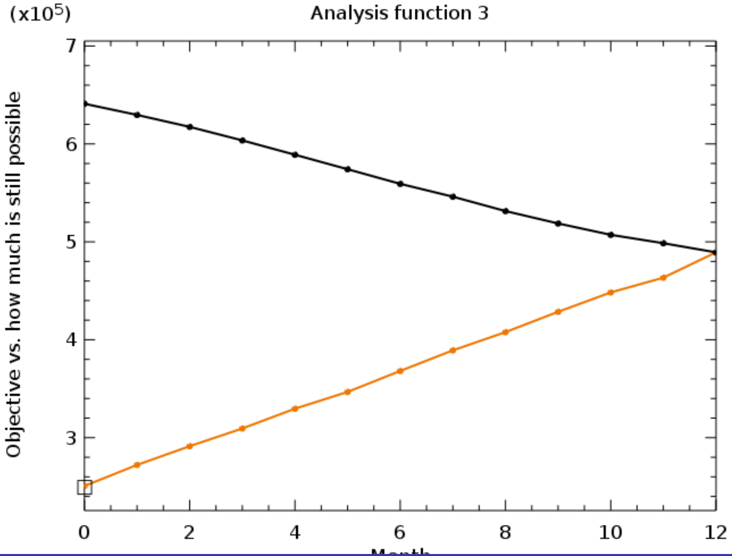
Objective of proband vs. potential (in black)



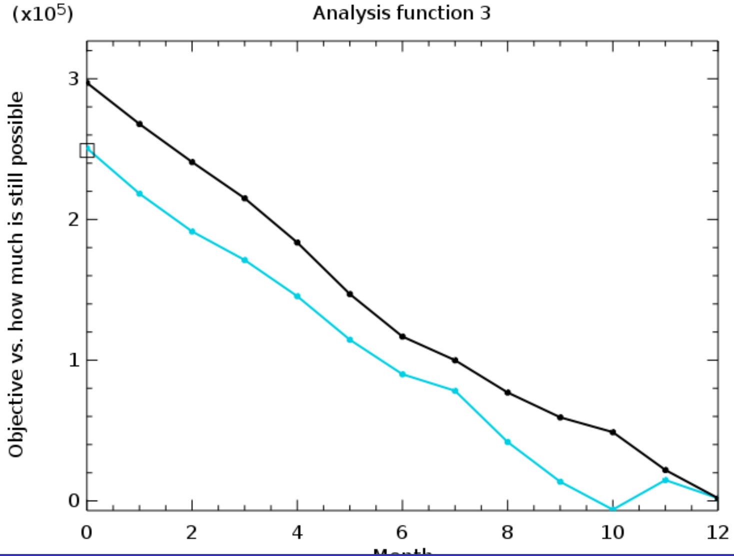
Objective of proband vs. potential (in black)



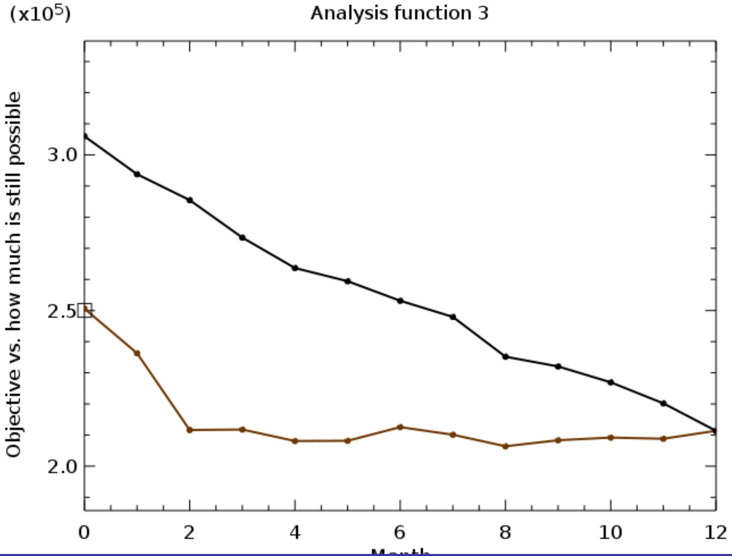
Objective of proband vs. potential (in black)



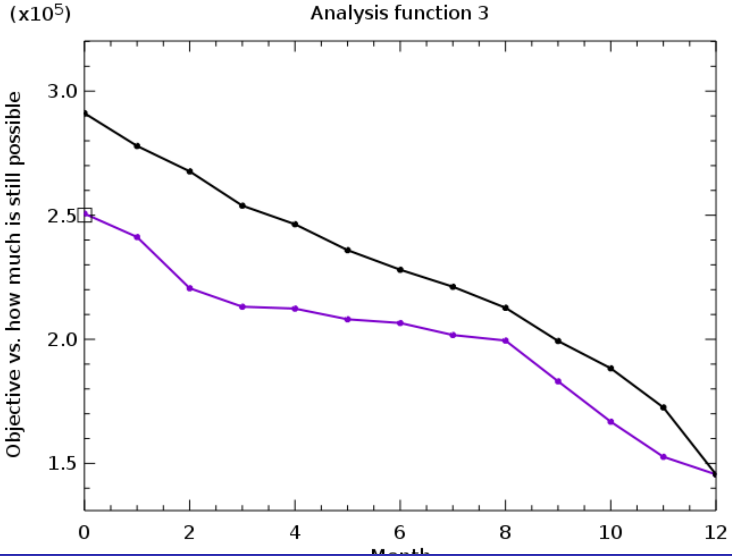
Objective of proband vs. potential (in black)



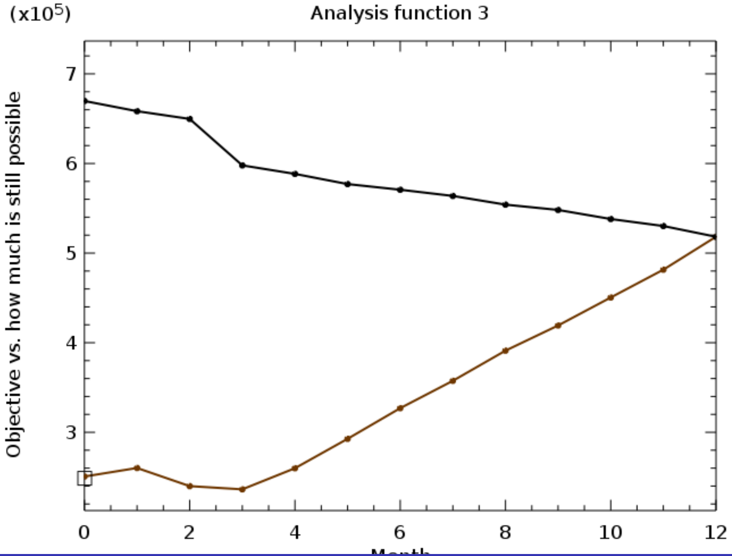
Objective of proband vs. potential (in black)



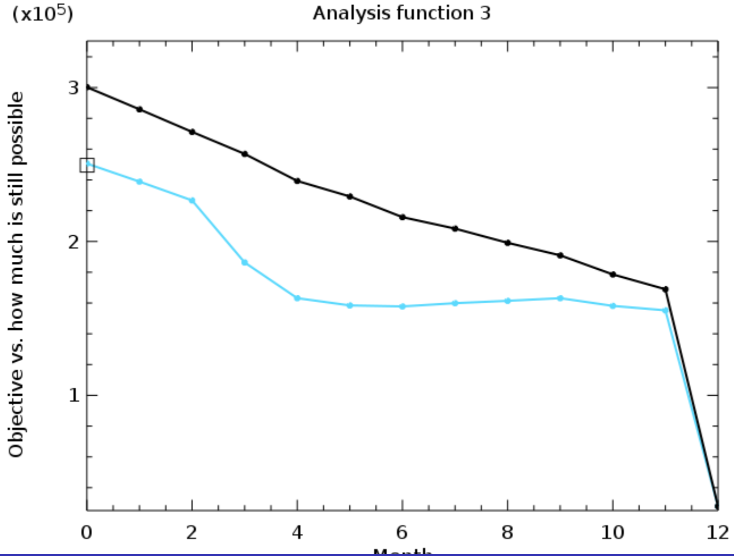
Objective of proband vs. potential (in black)



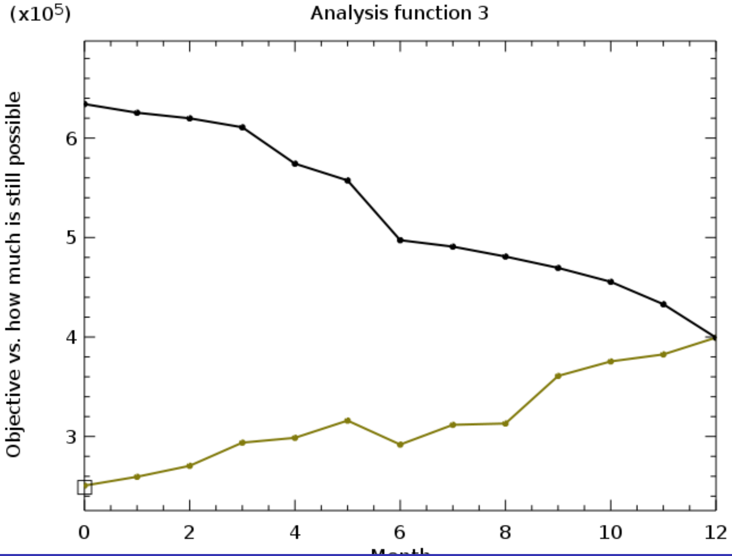
Objective of proband vs. potential (in black)



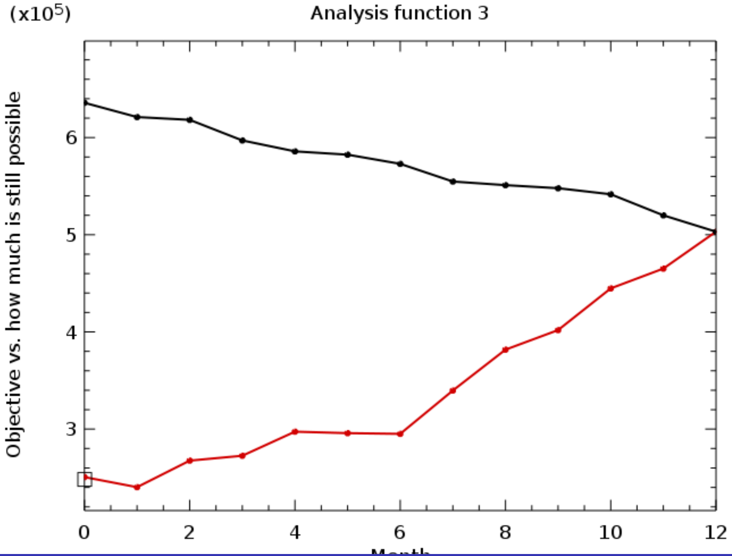
Objective of proband vs. potential (in black)



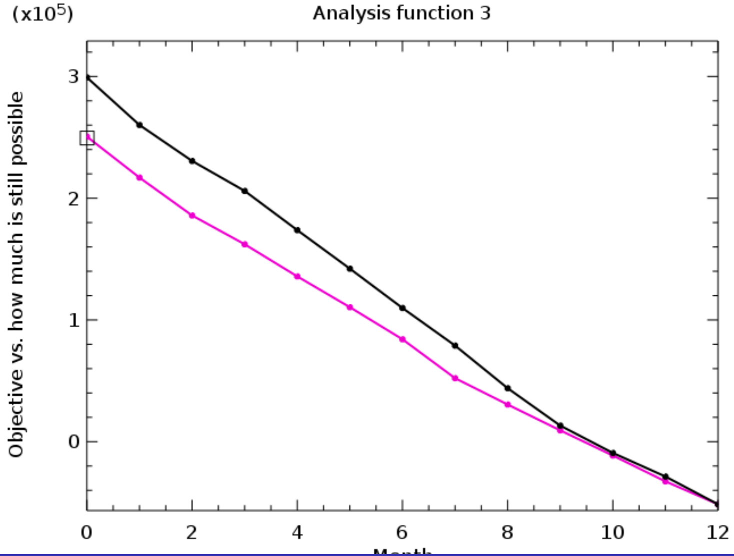
Objective of proband vs. potential (in black)



Objective of proband vs. potential (in black)



Objective of proband vs. potential (in black)



Further analysis

- Determine WHICH decision was really bad
- Can evaluate the derivative
- No need: already know the optimal solution
  - Look at $(u^*, x^*) - (u^{\mathrm{p}}, x^{\mathrm{p}})$

Further analysis

- Determine WHICH decision was really bad
- Can evaluate the derivative
- No need: already know the optimal solution
  - Look at $(u^*, x^*) - (u^\mathrm{p}, x^\mathrm{p})$

- Better:
  - Solve problem from $k+1$ to N as before
  - Add constraints $u_{k,i} = u^\mathrm{p}_{k,i}$, calculate Lagrange multipliers
  - Shadow prices: how much does decision i at time k cost?

Solver

- Modeling done with AMPL
- Automatization of interfaces

Solver

- Modeling done with AMPL
- Automatization of interfaces
- Structure exploiting interior point method
- IPOPT (Wächter et al.)
- Bonmin (Bonami et al.)

Solver

- Modeling done with AMPL
- Automatization of interfaces
- Structure exploiting interior point method
- IPOPT (Wächter et al.)
- Bonmin (Bonami et al.)

- Needed to solve $80 \cdot 12$ optimization problems
- Runtimes each on notebook
  - relaxed: < 1 sec.
  - integer: ≈ 3 min.

Solver

- Modeling done with AMPL
- Automatization of interfaces
- Structure exploiting interior point method
- IPOPT (Wächter et al.)
- Bonmin (Bonami et al.)

- Needed to solve $80 \cdot 12$ optimization problems
- Runtimes each on notebook
  - relaxed: < 1 sec.
  - integer: ≈ 3 min.
- Without hotstarts or advanced numerical techniques
- No multiple local minima found so far

Conclusions

- Computer based micro worlds used to understand human complex problem solving
- Modelled one of the most famous ones (tailorshop) as an optimization problem
- By solving series of optimization problems get valuable additional information
- Important: good modelling, exploiting structure

Outlook

- Apply new analysis tool to interesting test sets
- Apply statistical tools

Outlook

- Apply new analysis tool to interesting test sets
- Apply statistical tools
- Improve numerics
  - Warmstarts
  - Initial value embedding
- Will allow for online feedback

Outlook

- Apply new analysis tool to interesting test sets
- Apply statistical tools
- Improve numerics
  - Warmstarts
  - Initial value embedding
- Will allow for online feedback
- Combine analysis with investigation of human abstraction / simplification

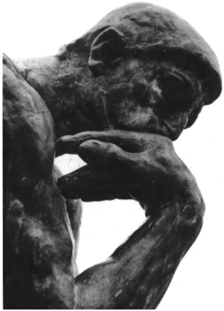
Outlook

- Apply new analysis tool to interesting test sets
- Apply statistical tools
- Improve numerics
  - Warmstarts
  - Initial value embedding
- Will allow for online feedback
- Combine analysis with investigation of human abstraction / simplification

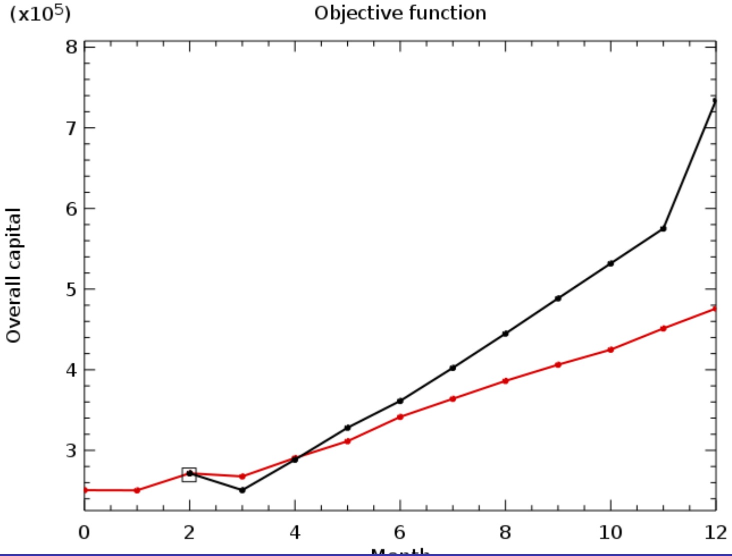
- Cite Joachim Funke: *From my point of view this is a sensational breakthrough in psychology.*
  This new analysis tool will revolutionize the research field!

Thank you very much for your attention!

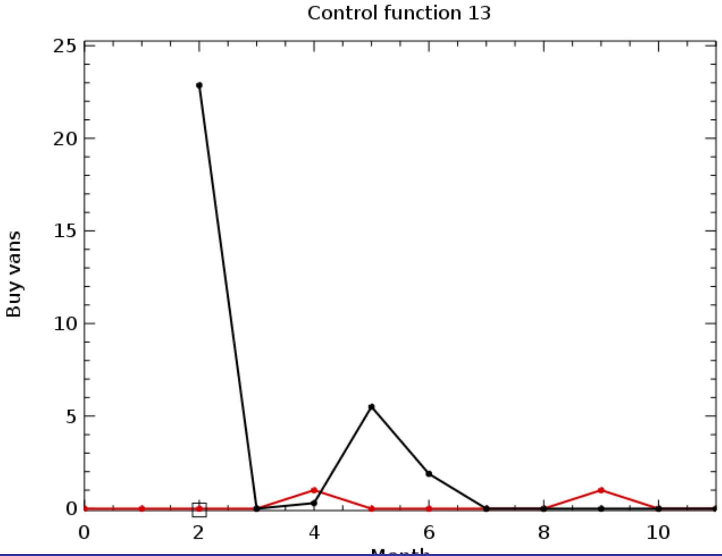
Questions as complex problems for me?



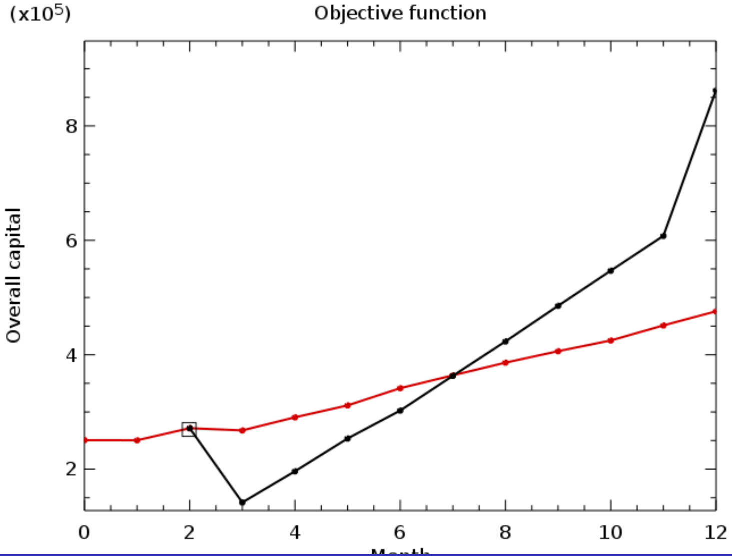
Add constraint: capital ≥ 0



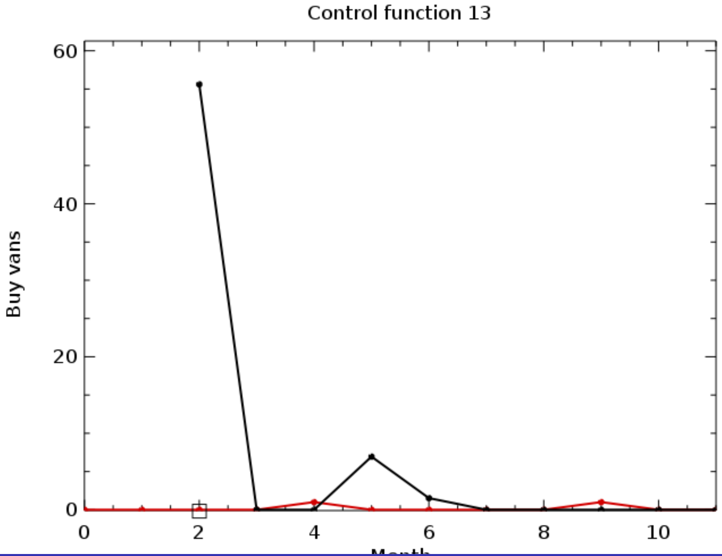
Add constraint: capital ≥ 0



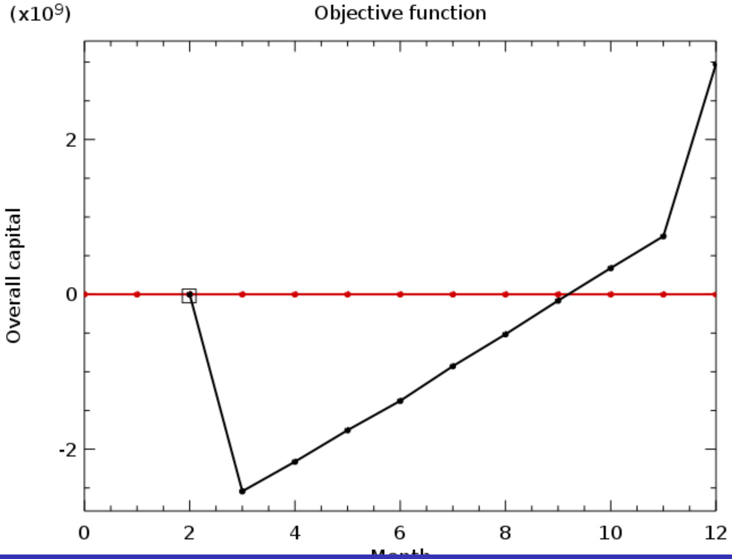
Add constraint: capital $\geq$ min capital of probands



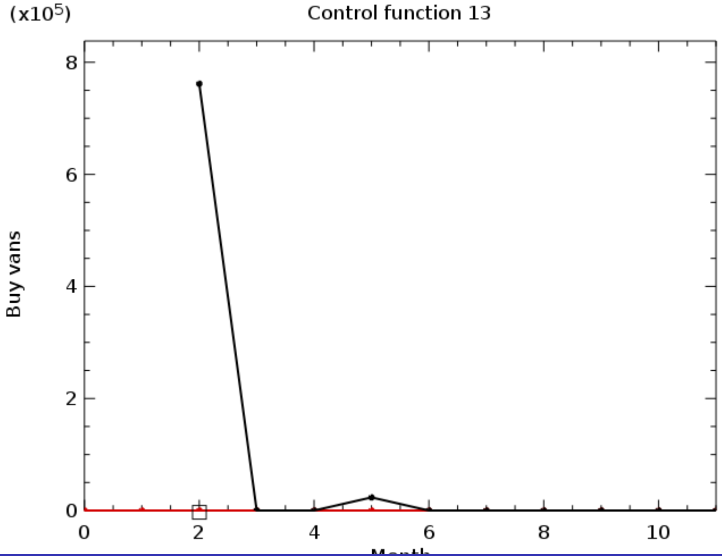
Add constraint: capital $\geq$ min capital of probands



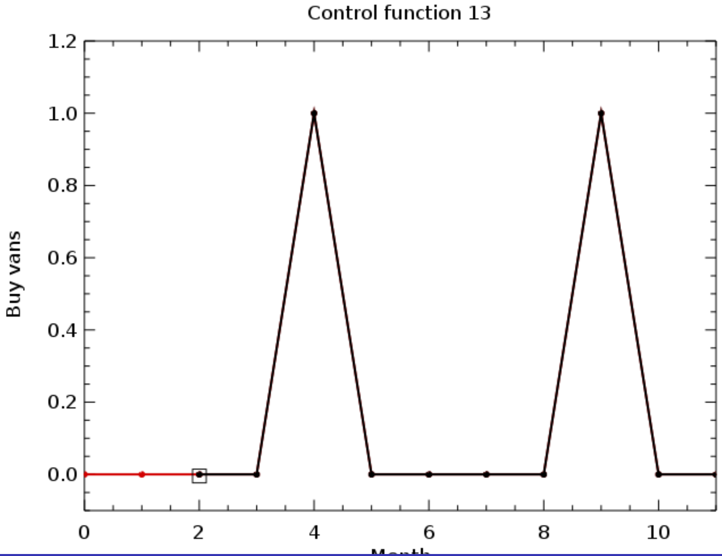
Add constraint: capital $\geq -10^{10}$



Add constraint: capital $\geq -10^{10}$



Fix # of vans to proband's choice



Fix # of vans to proband's choice